

A circular collage of mathematical symbols and formulas, arranged around a central point. The collage is divided into segments by radial lines, each containing different mathematical content. The segments are labeled with degrees: 0, 30, 60, 90, 120, 150, 180, 210, 240, 270, 300, and 330. The formulas include:
 - Trigonometric functions: $\sin(x)$, $\cos(x)$, $\tan(x)$, $\cot(x)$, $\sec(x)$, $\csc(x)$, $\arcsin(x)$, $\arccos(x)$, $\arctan(x)$, $\text{arccot}(x)$, $\text{arcsec}(x)$, $\text{arccsc}(x)$.
 - Algebraic equations: $x^2 + y^2 = r^2$, $x^2 + y^2 = 1$, $x^2 + y^2 = 4$, $x^2 + y^2 = 9$, $x^2 + y^2 = 16$, $x^2 + y^2 = 25$, $x^2 + y^2 = 36$, $x^2 + y^2 = 49$, $x^2 + y^2 = 64$, $x^2 + y^2 = 81$, $x^2 + y^2 = 100$.
 - Calculus: $\frac{d}{dx} \sin(x) = \cos(x)$, $\frac{d}{dx} \cos(x) = -\sin(x)$, $\frac{d}{dx} \tan(x) = \sec^2(x)$, $\frac{d}{dx} \cot(x) = -\csc^2(x)$, $\frac{d}{dx} \sec(x) = \sec(x)\tan(x)$, $\frac{d}{dx} \csc(x) = -\csc(x)\cot(x)$.
 - Geometry: $A = \pi r^2$, $V = \frac{4}{3}\pi r^3$, $C = 2\pi r$, $S = 4\pi r^2$, $A = \frac{1}{2}bh$, $A = \frac{1}{2}ab\sin(C)$, $A = \frac{1}{2}a^2 \sin(2\theta)$.
 - Other symbols: ∞ , $\frac{1}{x}$, $\frac{1}{y}$, $\frac{1}{z}$, $\frac{1}{w}$, $\frac{1}{v}$, $\frac{1}{u}$, $\frac{1}{t}$, $\frac{1}{s}$, $\frac{1}{r}$, $\frac{1}{q}$, $\frac{1}{p}$, $\frac{1}{o}$, $\frac{1}{n}$, $\frac{1}{m}$, $\frac{1}{l}$, $\frac{1}{k}$, $\frac{1}{j}$, $\frac{1}{i}$, $\frac{1}{h}$, $\frac{1}{g}$, $\frac{1}{f}$, $\frac{1}{e}$, $\frac{1}{d}$, $\frac{1}{c}$, $\frac{1}{b}$, $\frac{1}{a}$.

For exams in 2026 & onwards

INTRODUCTION TO ZUEB

The Ziauddin University Examination Board (ZUEB) is not only an awarding body but also a solution-driven educational organization dedicated to upholding the highest standards of academic excellence. ZUEB believes in excellence, integrity, and innovation in education. Established with a vision to foster a robust educational environment, ZUEB is committed to nurturing intellectual growth and development that meets international standards in an effective manner. The Ziauddin University Examination Board (ZUEB) was established through the Government Gazette No. XLI on June 6th, 2018. Its purpose is to ensure high quality, maintain global standards, and align the syllabi with national integrity within Pakistan's examination system. ZUEB manages student appeals, regulates assessments, and reviews policies to maintain high standards.

WHY CHOOSE HSSC-A AT ZUEB?

Ziauddin University Examination Board (ZUEB) offers the HSSC-A (Higher Secondary School Certificate Advance) program, designed for students from international educational backgrounds. This program provides a structured, affordable, and academically strong pathway for learners to align with Pakistan's education system. It allows students to fulfil national curriculum requirements, including Urdu, Islamiyat, Pakistan Studies, or Sindhi, with academic integrity and flexible learning options. ZUEB believes no student should be left behind due to financial limitations or cross-system transitions, and HSSC-A serves as a bridge between past efforts and future ambitions. It is the trusted choice for higher education in Pakistan

HSSC-ADVANCE MATHEMATICS

HSSC-Advance Mathematics at ZUEB is a foundation for exploring the language of logic and numbers, designed for students aspiring to pursue higher education in engineering, computer science, economics, actuarial sciences, and pure or applied mathematics. The course offers a rigorous, concept-driven curriculum aligned with both national and international standards, covering key topics such as algebra, calculus, geometry, trigonometry, probability, and statistics. Students develop a strong grasp of mathematical concepts and practical applications, while enhancing their problem-solving, logical reasoning, and analytical skills, ensuring they are both examination-ready and future-ready.

Aligned with national and international standards, HSSC-Advance Mathematics at ZUEB equips students with a comprehensive understanding of mathematical theories, methods, and applications that are essential for scientific and technological advancement. Designed for students aiming for careers in engineering, data science, finance, and academia, the course builds essential skills in quantitative analysis, abstract reasoning, and critical thinking.

Whether you are preparing for admission into top universities in engineering, science, or commerce, or planning a career in research, technology, or applied mathematics, HSSC-Advance Mathematics ensures you are academically prepared and nationally aligned, with a flexible, student-focused learning approach. Explore more on what HSSC-Advance Mathematics offers [ZUEB HSSC-Advance Official Page](#).

Syllabus Overview

No.	Content	XII	XIII	Assessment Objectives	Exam
1	Pure Mathematics 1	✓	✓	AO1, AO2 and AO3	A combination of written exam papers (externally set and marked) XII Paper 1 • Structured questions covering Pure Mathematics (1 & 2) • Duration: 2 hours 30 minutes • Weightage: 66% Paper 2 • Structured questions covering Mechanics 1 • Duration: 1 hour 15 minutes • Weightage: 34% XIII Paper 3 • Structured questions covering Pure Mathematics (1,2 & 3) • Duration: 2 hours • Weightage: 50% Paper 4 • Structured questions covering Mechanics (1 & 2) alongwith Probability and Statistics • Duration: 2 hours • Weightage: 50%
2	Pure Mathematics 2	✓	✓	AO1, AO2 and AO3	
3	Pure Mathematics 3	-	✓	AO1, AO2 and AO3	
4	Mechanics 1	✓	✓	AO1, AO2 and AO3	
5	Mechanics 2	-	✓	AO1, AO2 and AO3	
6	Probability and Statistics	-	✓	AO1, AO2 and AO3	

Description of Assessment Objectives

AO1 – Use and Apply Standard Techniques

- Select and accurately perform routine procedures
- Recall facts, terminology, and definitions with precision

AO2 – Reason, Interpret, and Communicate Mathematically

- Develop rigorous mathematical arguments, including proofs
- Make logical deductions and inferences
- Judge the validity of mathematical reasoning
- Clearly explain processes and reasoning
- Use appropriate mathematical language and notation correctly

AO3 – Problem Solving in Mathematics and Other Contexts

- Translate real-world and abstract problems into mathematical processes
- Interpret solutions in their original context, assessing accuracy and limitation
- Formulate mathematical models for contextual situations
- Apply and work with mathematical models
- Evaluate modelling outcomes, recognize model limitations, and, where needed, suggest refinements

Weighting of Assessment Objectives

Assessment Objectives	P1 (%)	P2 (%)	P3 (%)	P4 (%)
AO1	45	45	40	30
AO2	35	35	35	40
AO3	20	20	25	30

Pure Mathematics 1

Aim: To develop an understanding of mathematical notation and core concepts in algebra, functions, trigonometry, and calculus, and will apply these foundational skills to solve problems and interpret advanced mathematical relationships.

	The learner will:	SLO #	Assessment Criteria - The learner can:	Cognitive levels
1.1	Understanding algebraic expressions	1.1.1	Simplify and manipulate algebraic expressions by expanding brackets, collecting like terms, factorising , and rearranging equations to extract required information (e.g., solving for roots of polynomials).	AO2
		1.1.2	Interpret and apply the notation of positive and negative rational indices.	AO1
		1.1.3	Apply index laws to simplify expressions: $a^m \times a^n = a^{m+n}$ $a^m \div a^n = a^{m-n}$ $(a^m)^n = a^{mn}$ $(ab)^n = a^n b^n$	AO2
		1.1.4	Recognise surds as expressions containing non-rational roots.	AO1
		1.1.5	Simplify expressions using surd laws: $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$ $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$	AO2
		1.1.6	Identify conjugate pairs such as $(a + b)$ and $(a - b)$, and use the identity $(a + b)(a - b) = a^2 - b^2$.	AO1
		1.1.7	Rationalise denominators using conjugates, e.g. $\frac{1}{\sqrt{p}} \times \frac{\sqrt{p}}{\sqrt{p}} = \frac{\sqrt{p}}{p}$ $\frac{1}{\sqrt{p+q}} \times \frac{\sqrt{p+q}}{\sqrt{p+q}} = \frac{\sqrt{p+q}}{p+q}$	AO2
		1.1.8	Present irrational numbers in simplest surd form rather than converting to decimals.	AO1

1.2	Understanding quadratic equations	1.2.1	Recognise the general form of a quadratic equation in one variable, identifying the role of each constant and the condition that the leading coefficient is non-zero.	AO1
		1.2.2	Factorise quadratic expressions and determine solutions directly from the factors.	AO2
		1.2.3	Identify the number of possible real roots of a quadratic equation, distinguish between distinct and repeated roots, and classify the solutions accordingly.	AO3
		1.2.4	Complete the square to transform a quadratic equation into standard square form $a(x + b)^2 + c$, and extract solutions from this representation as $x = -b \pm \sqrt{-\frac{c}{a}}$.	AO2
		1.2.5	Relate the completed-square form $a(x + b)^2 + c$ of a quadratic equation to its graphical representation by locating the turning point (vertex) as $(-b, c)$.	AO2
		1.2.6	Apply the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve equations, and analyse the discriminant $b^2 - 4ac$ to determine the number and nature of the real roots.	AO3
		1.2.7	Solve quadratic equations and inequalities using appropriate methods such as factorisation, completing the square, or the quadratic formula, and justify the choice of method where appropriate.	AO3
		1.2.8	Recognise equations that are quadratic in form but involve a function of the variable (e.g $x = t$, $x = \sin(t)$ etc), and apply quadratic-solving methods to determine their solutions.	AO2
		1.2.9	Solve simultaneous equations involving one quadratic and one linear equation through substitution, and interpret the solutions in context.	AO3
		1.2.10	Sketch the graphs of quadratic functions, identify their key features such as turning points and intercepts, and interpret the relationship between the algebraic form and the graph.	AO3
1.3	Understanding functions	1.3.1	Use the standard symbols and classify numbers into real, integer, natural, and rational sets.	AO2
		1.3.2	Describe a map as a transformation from one set to another and illustrate how inputs correspond to outputs.	AO2

		1.3.3	Differentiate between one-to-one, many-to-one, and one-to-many mappings, and categorise them as onto or not onto.	AO3
		1.3.4	Identify the domain and range of a given mapping and relate them to the transformation process.	AO2
		1.3.5	Characterise a function as a mapping that is one-to-one or many-to-one, and recognise how this differs from other mappings.	AO2
		1.3.6	Analyse the properties of a map, including domain and range, and evaluate whether it qualifies as a function.	AO3
		1.3.7	Apply function composition using $f \circ g(x)$ or $f(g(x))$, and solve problems by combining two or more functions.	AO2
		1.3.8	Sketch and interpret transformations of basic functions of the form $y = g(x)$: $y = g(x) + c$ — vertical shift up by c units. $y = g(x + c)$ — horizontal shift left by c units. $y = cg(x)$ — vertical dilation (scale factor c). $y = g(cx)$ — horizontal dilation with scale factor $\frac{1}{c}$ $y = -g(x)$ — reflection across the x -axis. $y = g(-x)$ — reflection across the y -axis.	AO3
		1.3.9	Apply the properties of inverse of an one-to-one function $g(x)$: $gg^{-1}(x) = g^{-1}g(x) = x$ The graph of $g^{-1}(x)$ is the reflection of $g(x)$ on the line $y = x$ Domain of $g(x) = \text{Range of } g^{-1}(x)$ Range of $g(x) = \text{Domain of } g^{-1}(x)$	AO3
		1.3.10	Determine the inverse of a given function state its domain and range, and sketch the function together with its inverse and the line $y = x$.	AO3
1.4	Understanding coordinate geometry	1.4.1	Identify the slope-intercept form of the equation of a straight line as $y = mx + c$, where m is the slope and c is the y -intercept.	AO1
		1.4.2	Recognise and use other forms of the equation of the line: General form $\rightarrow ax + by + c = 0$ Slope-point form $\rightarrow (y - y_1) = m(x - x_1)$	AO2
		1.4.3	Determine whether three points are collinear by establishing whether they lie on the same straight line.	AO3

		1.4.4	Construct the equation of a straight line from two given points (x_1, y_1) and (x_2, y_2) by calculating the gradient as $m = \frac{y_2 - y_1}{x_2 - x_1}$ and intercept as $c = y_1 - mx_1$	AO2
		1.4.5	Formulate the equation of a straight line given a point (x_1, y_1) and a gradient m by using the slope-point form: $(y - y_1) = m(x - x_1)$	AO2
		1.4.6	Determine whether two straight lines are parallel, perpendicular, or neither, using their slopes.	AO3
		1.4.7	Calculate the distance between two points on a line, determine the midpoint between them, and find the bounded area under a straight-line graph between two vertical limits $x = a$ and $x = b$.	AO2
		1.4.8	Determine the midpoint of a line segment between two given points.	AO2
		1.4.9	Compute the area enclosed by a straight line, the x-axis, and two vertical boundaries.	AO2
		1.4.10	Find the point of intersection between two straight lines by solving their simultaneous equations.	AO2
		1.4.11	Construct the equation of the perpendicular bisector of a line segment.	AO2
		1.4.12	Recall and use the standard equation of a circle $x^2 + y^2 + 2\alpha x + 2\beta y + \gamma = 0$ with its centre at $(-\alpha, -\beta)$ and radius given by $\sqrt{\alpha^2 + \beta^2 - \gamma}$.	AO2
		1.4.13	Recognise alternate forms of the circle equation and determine the centre and radius from them.	AO2
		1.4.14	Determine the points of intersection between a circle and a straight line by solving their simultaneous equations.	AO3
		1.4.15	Construct the equation of a tangent to a circle at a specified point.	AO2
		1.4.16	Find the perpendicular bisector of a chord of a circle and use it to reason about circle properties.	AO3
		1.4.17	Determine the points of intersection of a quadratic curve and a straight line, and deduce the corresponding quadratic solutions.	AO3

1.5	Understanding angles and trigonometry	1.5.1	Interpret the definition of a radian as a unit of angular measure and convert angles between degrees and radians.	AO2
		1.5.2	Recall and use the exact degree–radian conversions for common angles such as 30° , 45° , 60° , 90° , 180° , and 360° .	AO2
		1.5.3	Apply the formulas for arc length ($s = r\theta$) and sector area ($A = \frac{1}{2}r^2\theta$) to solve problems involving portions of a circle.	AO2
		1.5.4	Sketch and use the graphs of sine, cosine, and tangent functions for angles expressed in either degrees or radians.	AO2
		1.5.5	Sketch and interpret graphs of simple transformations of the sine, cosine, and tangent functions.	AO3
		1.5.6	Recall the exact values of sine, cosine, and tangent for 30° , 45° , 60° , and related angles, and apply them in solving problems.	AO2
		1.5.7	Use the inverse trigonometric functions as $\sin^{-1}x$, $\cos^{-1}x$ and $\tan^{-1}x$ to determine unknown angles within their principal ranges, extend solutions using trigonometric properties, and justify the appropriate choice of solution	AO3
		1.5.8	Apply fundamental trigonometric identities, such as the Pythagorean identity ($\sin^2\theta + \cos^2\theta \equiv 1$) and the relationship between tangent, sine, and cosine ($\frac{\sin\theta}{\cos\theta} \equiv \tan\theta$) to simplify expressions and prove further identities.	AO3
		1.5.9	Determine all solutions of trigonometric equations within a specified interval, using knowledge of periodicity and symmetry.	AO3
		1.5.10	Solve right-angled triangle problems by applying trigonometric ratios to calculate unknown sides or angles.	AO2
		1.5.11	Apply the sine rule ($\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$) and cosine rule ($a^2 = b^2 + c^2 - 2(bc)\cos A$) to find missing sides or angles in non-right-angled triangles.	AO2
		1.5.12	Calculate the area of a triangle using standard geometric methods or the formula involving sine of an angle between two sides: $A = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}ab\sin C$).	AO2

		1.5.13	Determine the area of a circular segment by applying geometric reasoning and trigonometric methods.	AO3
1.6	Understanding differentiation	1.6.1	Interpret differentiation as the limiting process of finding the gradient of a chord as its length approaches zero.	AO1
		1.6.2	Describe the derivative of a function at a point as the gradient there, and generalise the derivative as the gradient function across all points.	AO1
		1.6.3	Recognise and use the standard notation for limits $\lim_{h \rightarrow 0}$ the context of differentiation.	AO2
		1.6.4	Use different notations for derivatives: $\frac{d}{dx} \rightarrow$ derivative with respect to x $\frac{d}{dx} f(x) = \frac{dy}{dx} = f'(x) = y'$	AO2
		1.6.5	Differentiate power functions of the form x^n where $n \neq 0$.	AO2
		1.6.6	Differentiate functions with multiple terms by applying differentiation rules term by term: If $y = u + v$, then $y' = u' + v'$.	AO2
		1.6.7	Apply the rule for differentiating constant multiples of functions and polynomials. $\frac{d}{dx} cf(x) = c \frac{d}{dx} f(x)$	AO2
		1.6.8	Construct the equations of tangents and normal on the function $f(x)$ at point $x = a$ by using the following equation: Equation of tangent $\rightarrow y - f(a) = f'(a)(x - a)$ Equation of normal $\rightarrow y - f(a) = -\frac{1}{f'(a)}(x - a)$	AO3
		1.6.9	Interpret the second derivative as the rate of change of the gradient function.	AO3
		1.6.10	Use different notations for the second derivative and express it equivalently as the derivative of the derivative: $\frac{d^2}{dx^2} \rightarrow$ second derivative with respect to x $\frac{d}{dx} f'(x) = \frac{d^2 y}{d^2 x} = f''(x) = y''$	AO2

1.7	Understanding integration	1.7.1	Recognise integration as the inverse process of differentiation, identify the integrand, and understand the role of the constant of integration.	AO1
		1.7.2	Apply the concept of an indefinite integral of $f'(x)$ as the family of functions that differentiate to $f'(x)$, and use standard integral $\int f'(x)dx = f(x) + c$ notation correctly.	AO2
		1.7.3	Integrate power functions of the form x^n where $n \neq -1$.	AO2
		1.7.4	Integrate functions with multiple terms by applying the rule of term-by-term integration: $\int f'(x) + g'(x) dx = f(x) + g(x) + c$	AO2
		1.7.5	Integrate polynomial functions accurately using standard rules.	AO2
		1.7.6	Determine the constant of integration using given conditions, such as a point through which the integral curve passes.	AO3

Pure Mathematics 2

Aim: Pure Mathematics 2 aims to build on the knowledge developed in Pure Mathematics 1, extending into more advanced concepts. It deepens understanding of algebra, trigonometry, and calculus, while introducing new areas such as exponentials, logarithms, sequences and series, and numerical methods for solving equations. Learners are expected to retain and apply prior Pure Mathematics 1 content, as it remains integral to assessment in this unit.

	The learner will:	SLO #	Assessment Criteria - The learner can:	Cognitive levels
2.1	Understanding algebra	2.1.1	Recall and interpret the meaning of the modulus (absolute value) function $ f(x) $, where the argument is given by $f(x)$.	AO1
		2.1.2	Construct and sketch graphs of modulus functions of the form $(y = ax + b)$ for linear functions and $y = f(x)$ for non-linear functions.	AO2
		2.1.3	Determine the solutions of equations involving modulus functions.	AO2
		2.1.4	Manipulate and apply relationships: $ p = q \Leftrightarrow p^2 = q^2$ $ x - p < q \Leftrightarrow p - q < x < p + q$ to solve equations and inequalities.	AO3
		2.1.5	Perform long division of polynomials (up to degree 4) and factorisation, clearly identifying quotient and remainder.	AO2
		2.1.6	State and interpret the Factor Theorem for the polynomial $p(x)$ for the following cases: $p(q) = 0 \Leftrightarrow (x - q)$ is a factor of $p(x)$. $p(\frac{q}{r}) = 0 \Leftrightarrow (rx - q)$ is a factor of $p(x)$.	AO1
		2.1.7	Apply the Factor Theorem to factorise higher-order polynomials.	AO2
		2.1.8	State and interpret the Remainder Theorem for polynomials divided by linear factors: If $p(x)$ is divided by $(rx - q)$, then the remainder is $p(\frac{q}{r})$.	AO1

		2.1.9	Solve polynomial equations and rational equations involving quotients of polynomials.	AO3
2.2	Understanding logarithms and exponentials.	2.2.1	Interpret and apply the definitions of logarithmic and exponential functions, and demonstrate their connection as inverse functions.	AO2
		2.2.2	Relate the natural logarithm to the mathematical constant e and use this link in problem solving.	AO2
		2.2.3	Apply the laws of logarithms: Multiplication Law: $\log_a p + \log_a q = \log_a pq$ Division Law: $\log_a p - \log_a q = \log_a \frac{p}{q}$ Power Law: $\log_a (p)^n = n \log_a p$ And the special use cases: $\log_a \left(\frac{1}{p} \right) = -\log_a p$ $\log_p p = 1, \text{ when } p > 0 \text{ and } p \neq 1$ $\log_a 1 = 0 \text{ when } a > 0 \text{ and } a \neq 1$	AO3
		2.2.4	Transform logarithmic expressions by changing their base and justify the equivalence of different forms. $\log_a (x) = \frac{\log_b (x)}{\log_b (a)}$ $\log_a (b) = \frac{1}{\log_b (a)}$	AO3
		2.2.5	Sketch graphs of exponential functions (e^{ks}) with positive and negative growth rates (k positive and negative respectively), and the natural logarithm function (logarithm with the base e).	AO2
		2.2.6	Solve equations and inequalities where the variable appears in the exponent using appropriate techniques.	AO3
		2.2.7	Transform exponential equations into linear form using logarithms, and solve by interpreting the gradient and intercept.	AO3

2.3	Understanding sequences and series	2.3.1	Represent a series of terms using sigma notation $\sum_{i=a}^b f_i(x)$ and evaluate simple summations.	AO2
		2.3.2	Apply Pascal's Triangle to expand binomial expressions of the form $(a + b)^n$.	AO2
		2.3.3	Recall and use the definition of factorial notation, including special cases such as $0! = 1$.	AO2
		2.3.4	Compute combinations using the formula for nCr : $nCr = \frac{n!}{r!(n-r)!}$ and interpret their use in counting problems.	AO3
		2.3.5	Determine the r-th entry in the n-th row of Pascal's Triangle using $\frac{(n-1)!}{(r-1)!(n-r)!}$.	AO2
		2.3.6	Expand expressions of the form $(a + b)^n$ for $n \leq 5$ using the Binomial Theorem: $(a + b)^n = \sum_{k=0}^n \frac{n!}{k!(n-k)!} a^k b^{n-k}.$	AO2
		2.3.7	Approximate large powers using truncated binomial expansions, and estimate values when $0 < m < 1$.	AO3
		2.3.8	Identify arithmetic and geometric sequences, and calculate the n-th term as well as the sum of the first n terms.	AO3
		2.3.9	Determine whether a geometric series is convergent and calculate its sum to infinity where applicable.	AO3
2.4	Understanding trigonometry	2.4.1	Solve trigonometric equations of the form $\sin(n\theta) = a, \cos(n\theta) = a$, and $\tan(n\theta) = b$ within a specified interval and know the ranges of their outputs ($-1 \leq a \leq 1$ and $-\infty \leq b \leq \infty$).	AO2
		2.4.2	Solve trigonometric equations of the form $\sin(\theta + \alpha) = a, \cos(\theta + \alpha) = a$, and $\tan(\theta + \alpha) = b$ within a given interval and know the ranges of their outputs ($-1 \leq a \leq 1$ and $-\infty \leq b \leq \infty$).	AO2
		2.4.3	Recall and use the definitions of reciprocal trigonometric functions (<i>sec</i> , <i>cosec</i> , <i>cot</i>) and determine the values for which they are undefined.	AO2

		2.4.4	Sketch and interpret the graphs of <i>sec</i> , <i>cosec</i> , and <i>cot</i> in degrees and radians.	AO3
		2.4.5	Apply the addition formulae to solve trigonometric equations and prove identities. $\sin P + \sin Q = 2 \sin\left(\frac{P+Q}{2}\right) \cos\left(\frac{P-Q}{2}\right)$ $\sin P - \sin Q = 2 \cos\left(\frac{P+Q}{2}\right) \sin\left(\frac{P-Q}{2}\right)$ $\cos P + \cos Q = 2 \cos\left(\frac{P+Q}{2}\right) \cos\left(\frac{P-Q}{2}\right)$ $\cos P - \cos Q = 2 \sin\left(\frac{P+Q}{2}\right) \sin\left(\frac{P-Q}{2}\right)$	AO3
		2.4.6	Use compound-angle formulae to solve trigonometric equations and prove identities: $\sin(P \pm Q) = \sin P \cos Q \pm \cos P \sin Q$ $\cos(P \pm Q) = \cos P \cos Q \mp \sin P \sin Q$ $\tan(P \pm Q) = \frac{\tan P \pm \tan Q}{1 \mp \tan P \tan Q} \text{ where } (P \pm Q) \neq (n + \frac{1}{2})\pi$	AO3
		2.4.7	Derive and use the double-angle formulae from addition formulae.	AO2
		2.4.8	Express sums of sine and cosine functions in the form of a single trigonometric function using R and α : $a \sin x \pm b \cos x = R \sin(x \pm \alpha)$ $a \cos x \pm b \sin x = R \cos(x \mp \alpha)$ Where $R > 0$, $0 < \alpha < \frac{\pi}{2}$, $R \cos \alpha = a$, $R \sin \alpha = b$ and $R = \sqrt{a^2 + b^2}$.	AO3
		2.4.9	Apply reciprocal trigonometric identities to solve equations and prove identities: $\sec^2 \theta \equiv 1 + \tan^2 \theta$ $\operatorname{cosec}^2 \theta \equiv 1 + \cot^2 \theta$	AO3
2.5	Understanding differentiation	2.5.1	Identify that a function $f(x)$ is increasing over an interval $[p, q]$ if $f'(x) \geq 0$ for all values of x such that $p < x < q$. It is strictly increasing if $f'(x) > 0$ for all x such that $p < x < q$.	AO2
		2.5.2	Identify that a function $f(x)$ is decreasing over an interval $[p, q]$ if $f'(x) \leq 0$ for all values of x such that $p < x < q$. It is strictly decreasing if $f'(x) < 0$ for all x such that $p < x < q$.	AO2
		2.5.3	Identify that a function $f(x)$ is unchanging over an interval $[p, q]$ if $f'(x) = 0$ for all values of x such that $p < x < q$	AO1

		2.5.4	Determine whether a function is increasing, decreasing, or constant over a region, and calculate the specific values of x where these conditions hold.	AO2
		2.5.5	Find the stationary points of a function by solving where the gradient equals zero.	AO2
		2.5.6	Classify a stationary point as a local maximum, local minimum, or point of inflection using the second derivative test.	AO3
		2.5.7	Classify a stationary point $x = a$ when the second derivative is zero ($f''(a) = 0$), by testing the sign of the first derivative on either side of the point.	AO3
		2.5.8	Recall and use the derivatives of standard functions including exponential (e^x), logarithmic ($\ln x$), sine ($\sin x$), cosine ($\cos x$), and tangent ($\tan x$).	AO2
		1.5.9	Compute derivatives of sums, differences, and constant multiples of functions.	AO2
		2.5.10	Sketch the graphs of functions, clearly showing the position and nature of stationary points.	AO3
		2.5.11	Sketch graphs of derivative (gradient) functions, highlighting the position and nature of stationary points.	AO3
		2.5.12	Solve contextual problems involving rates of change, where the derivative represents $\frac{\Delta y}{\Delta x}$.	AO3
2.6	Understanding integration	2.6.1	Recall and use the integrals of standard functions such as exponential (e^x), reciprocal ($\frac{1}{x}$), sine ($\sin x$), cosine ($\cos x$), and secant squared ($\sec^2 x$).	AO2
		2.6.2	Compute integrals involving sums, differences, and constant multiples of functions.	AO2
		2.6.3	Interpret the definite integral as the evaluation of an integral between two limits, and apply the notation to calculate results: $\int_a^b f'(x)dx = f(a) - f(b).$	AO2
		2.6.4	Evaluate definite integrals of functions with explicit and implicit limits.	AO2

		2.6.5	Relate the definite integral of a function to the area between the curve, the x-axis, and given vertical boundaries $x = a$ and $x = b$.	AO2
		2.6.6	Recognise that areas under the x -axis are negative and separate them appropriately in calculations.	AO2
		2.6.7	Determine areas enclosed by a curve and an axis using integration methods.	AO2
		2.6.8	Calculate the area between two functions using integration and/or geometric reasoning.	AO3
		2.6.9	Apply the Trapezium Rule to approximate the area under a curve that cannot be integrated analytically: $\int_a^b ydx = \frac{1}{2}h(y_0 + 2(y_1 + y_2 + y_3 + \dots + y_{n-1}) + y_n)$ Where the step size $h = \frac{b-a}{n}$ and $y_i = f(a + ih)$ and judge whether the estimate is greater or less than the true value.	AO3
2.7	Understanding umerical methods	2.7.1	Estimate the roots of a function using graphical methods.	AO2
		2.7.2	Detect a root of a continuous function by identifying a sign change over an interval.	AO3
		2.7.3	Approximate the solutions of a function using iteration.	AO2
		2.7.4	Use graphical methods to select a suitable initial value for which an iterative formula will converge.	AO3
		2.7.5	Recognise that for the same function, different iterative formulae may converge to different roots, and that some may diverge depending on the initial value.	AO3

Pure Mathematics 3

Aim: Pure Mathematics 3 aims to build upon the foundations of Pure Mathematics 2, extending into advanced algebraic techniques and deeper calculus applications. This unit introduces new areas such as vectors and complex numbers, while strengthening the understanding of mathematical proofs. Prior knowledge of Pure Mathematics 1 and 2 is assumed and assessed throughout, even though not explicitly listed here.

	The learner will:	SLO #	Assessment Criteria - The learner can:	Cognitive levels
3.1	Understanding Mathematical proofs	3.1.1	Identify the structure and understand the requirements of a valid mathematical proof, including the logical flow and necessary assumptions.	AO1
		3.1.2	Prove an identity by carrying out algebraic manipulation, and apply the correct symbol ($\equiv$) to denote equivalence (“is always equal to”).	AO2
		3.1.3	Construct mathematical arguments using Proof by Deduction, demonstrating a logical sequence from assumptions to conclusion.	AO3
		3.1.4	Apply Proof by Exhaustion to verify mathematical statements by examining all possible cases (where fewer than five cases are involved).	AO3
		3.1.5	Disprove mathematical statements by presenting counter-examples that invalidate a general claim. (Proof by counter-example)	AO3
		3.1.6	Use alternative proof strategies (e.g., Proof by Contradiction) to establish the validity of mathematical results.	AO3
3.2	Understanding partial fractions	3.2.1	Recognise that improper algebraic fractions occur when the degree of the numerator is greater than or equal to the degree of the denominator.	AO1
		3.2.2	Apply algebraic long division to rearrange an improper algebraic fraction into a mixed fraction form:	AO2

			$\frac{f(x)}{g(x)} = q(x) + \frac{r}{g(x)}$ where $q(x)$ is the quotient and r is the remainder obtained from long division.	
		3.2.3	Decompose a proper fraction with linear factors in the denominator (up to three factors) into partial fractions with unknown constants: $\frac{f(x)}{(x+a)(x+b)(x+c)} = \frac{A}{x+a} + \frac{B}{x+b} + \frac{C}{x+c}$	AO2
		3.2.4	Decompose a proper fraction with linear factors, including one repeated factor (up to two factors), into partial fractions with the correct structure: $\frac{f(x)}{(x+a)(x+b)^2} = \frac{A}{x+a} + \frac{B}{x+b} + \frac{C}{(x+b)^2}$	AO2
		3.2.5	Decompose an improper fraction with linear factors in the denominator (up to two factors) into partial fractions, where the polynomial part matches the degree requirement. $\frac{f(x)}{(x+b)(x+c)} = A(x) + \frac{B}{(x+b)} + \frac{C}{(x+c)}$	AO2
3.3	Understanding differentiation	3.3.1	Differentiate products of functions using the Product Rule: $\frac{d}{dx}(u \cdot v) = v \cdot u' + u \cdot v'$	AO2
		3.3.2	Differentiate functions of functions using the Chain Rule: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$	AO2
		3.3.3	Apply the reciprocal case of the Chain Rule: $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ noting its limitation that $\frac{dx}{dy}$ is not a fraction but simply the reciprocal of $\frac{dy}{dx}$.	AO2
		3.3.4	Differentiate quotients of functions using the Quotient Rule. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot u' - u \cdot v'}{v^2}$	AO2
		3.3.5	Use the derivatives of the trigonometric functions sec, cot, and cosec (their derivatives are given in the formula sheet).	AO1

		3.3.6	Differentiate the inverse trigonometric functions $\sin^{-1}x$, $\cos^{-1}x$, and $\tan^{-1}x$.	AO2
		3.3.7	Apply parametric differentiation to find $\frac{dy}{dx}$ when $x = f(t)$, $y = g(t)$.	AO2
		3.3.8	Apply implicit differentiation to find $\frac{dy}{dx}$ from relations involving x and y .	AO2
3.4	Understanding integration	3.4.1	Use standard integrals and inverse derivatives to integrate functions.	AO2
		3.4.2	Apply the chain rule in reverse to integrate functions of the form $f(ax + b)$.	AO2
		3.4.3	Apply trigonometric identities to integrate functions.	AO2
		3.4.4	Solve integrals of the form $\int \frac{f'(x)}{f(x)} dx = \ln f(x) + c$.	AO2
		3.4.5	Solve integrals of the form $\int f'(x)(f(x))^n dx = \frac{f(x)^{n+1}}{n+1} + c$.	AO2
		3.4.6	Apply integration by parts when instructed: $\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$.	AO2
		3.4.7	Apply integration by substitution when given a substitution.	AO2
		3.4.8	Use partial fractions to integrate rational functions (when instructed).	AO2
3.5	Understand vectors	3.5.1	Define scalar and vector quantities, distinguishing between magnitude and direction.	AO1
		3.5.2	Recognise and use standard notation for scalars and vectors.	AO1
		3.5.3	Apply the fact that equal vectors represent parallel and equal line segments.	AO2

	3.5.4	Use vector equivalences: $\underline{AB} = \underline{BA}$, $\underline{AB} + \underline{BC} = \underline{AC}$, $\underline{AB} + \underline{BA} = 0$	AO2
	3.5.5	Apply the Parallelogram Law for vector addition.	AO2
	3.5.6	Represent and compute vectors in column form under addition and scalar multiplication.	AO2
	3.5.7	Represent and compute vectors in unit vector form $(\hat{i}, \hat{j}, \hat{k})$, including scalar multiplication and addition.	AO2
	3.5.8	Calculate the magnitude of a vector using $ v $.	
	3.5.9	Express vectors in magnitude–direction form, finding angle with positive x -axis using $\theta = \tan^{-1}(\frac{b}{a})$.	AO2
	3.5.10	Solve geometric problems in 2D using vector methods.	AO2
	3.5.11	Represent position vectors from origin to a point, and use them in vector differences: $\underline{AB} = \underline{OB} - \underline{OA}$	AO1
	3.5.12	Determine the position vector of a point dividing a line segment AB in the ratio $AP:PB$ by using: $\underline{OP} = \underline{OA} + \frac{a}{a+b}(\underline{OB} - \underline{OA})$	AO2
	3.5.13	Apply vector uniqueness in 2D: if $p\underline{a} + q\underline{b} = r\underline{a} + s\underline{b}$, then $p=r$, $q=s$.	AO2
	3.5.14	Recognise that vectors may have any number of dimensions.	AO1
	3.5.15	Calculate the magnitude 3D vector $v = (a, b, c)$ by using the formula $\sqrt{a^2 + b^2 + c^2}$ and find angles with coordinate axes using direction cosines: $\cos(\theta_x) = \frac{a}{ v }$ $\cos(\theta_y) = \frac{b}{ v }$ $\cos(\theta_z) = \frac{c}{ v }$	AO2

		3.5.16	Calculate the distance between two points $(x_1, x_2, x_3, \dots, x_n)$ and $(y_1, y_2, y_3, \dots, y_n)$ in n-dimensional space: $d = \sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2 + \dots + (y_n - x_n)^2}$	AO2
		3.5.17	Apply uniqueness of vector representation in 3D: if $p\mathbf{a} + q\mathbf{b} + r\mathbf{c} = p'\mathbf{a} + q'\mathbf{b} + r'\mathbf{c}$, then coefficients are equal.	AO2
		3.5.18	Formulate and use the vector equation of a straight line in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$.	AO2
		3.5.19	Formulate and use the vector equation of a line through two points $\mathbf{c}$ and $\mathbf{d}$: $\mathbf{r} = \mathbf{c} + \lambda(\mathbf{d} - \mathbf{c})$.	AO2
		3.5.20	Understand and apply the scalar (dot) product: $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \mathbf{b} \cos\theta$	AO2
		3.5.21	Interpret $\mathbf{a} \cdot \mathbf{b}$ as the magnitude of projection of one vector onto another.	AO2
		3.5.22	Determine perpendicularity of vectors using $\mathbf{a} \cdot \mathbf{b} = 0$.	AO2
		3.5.23	Compute the scalar product using components: $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$	AO2
		3.5.24	Find the point of intersection of two non-parallel, non-skew lines in 3D.	AO2
		3.5.25	Calculate the angle between two intersecting lines using: $\cos\theta = \left \frac{\mathbf{a} \cdot \mathbf{b}}{ \mathbf{a} \mathbf{b} } \right $	AO2
3.6	Understanding complex numbers	3.6.1	Recognise that $\sqrt{-1}$ has no real solutions, and introduce the imaginary unit $i = \sqrt{-1}$.	AO1
		3.6.2	Recognise that imaginary numbers are of the form bi , and complex numbers of the form $a + bi$.	AO1
		3.6.3	Identify the real part $\text{Re}(z)$ and the imaginary part $\text{Im}(z)$ of a complex number $z = a + bi$.	AO1
		3.6.4	Determine equality of two complex numbers by comparing real and imaginary parts.	AO2

	3.6.5	Perform addition and subtraction of complex numbers.	AO2
	3.6.6	Perform multiplication of a complex number by a real scalar.	AO2
	3.6.7	Perform multiplication of two complex numbers in the form $a + bi$.	AO2
	3.6.8	Use the complex conjugate $z^* = a - bi$ in calculations.	AO2
	3.6.9	Show and use that $z + z^* = 2a$ is real.	AO2
	3.6.10	Show and use that $z \cdot z^* = a^2 + b^2$ is real.	AO2
	3.6.11	Perform division of complex numbers in the form $\frac{a+bi}{c+di}$.	AO2
	3.6.12	Solve quadratic equations with complex roots when $b^2 - 4ac < 0$.	AO2
	3.6.13	Apply the conjugate root theorem for polynomials with real coefficients.	AO2
	3.6.14	Find all real and complex roots of a cubic equation, given one complex root.	AO2
	3.6.15	Find all real and complex roots of a quartic equation, given one complex root.	AO2

Mechanics 1

Aim: Develop the ability to apply mathematical methods to model and analyse real-world physical situations. Build understanding of how forces influence the motion of objects, the conditions for equilibrium, and the principles of energy transfer in mechanical systems. This topic also aims to strengthen links between mathematical reasoning and physical interpretation.

	The learner will:	SLO #	Assessment Criteria - The learner can:	Cognitive levels
4.1	Understanding statics	4.1.1	Recognise that forces are vectors and use vector notation to represent them.	AO1
		4.1.2	Construct free-body diagrams for simple systems with multiple forces.	AO2
		4.1.3	Resolve forces into orthogonal components (not limited to horizontal and vertical).	AO2
		4.1.4	Calculate the magnitude and direction of the resultant of parallel and orthogonal forces.	AO2
		4.1.5	Apply the condition of equilibrium: resultant force is zero and vector sum of components in any direction is zero.	AO2
		4.1.6	Determine the force (magnitude and/or direction) required to maintain equilibrium of a particle.	AO2
		4.1.7	Apply Newton's Third Law to describe the equal and opposite nature of interacting forces.	AO1
		4.1.8	Identify Newton's Third Law pairs and represent them in force diagrams for a system.	AO2
		4.1.9	Represent contact forces using normal and frictional components.	AO1
		4.1.10	Interpret the concept of limiting equilibrium and the associated terminology.	AO3
		4.1.11	Use the coefficient of limiting friction (μ) in equilibrium situations, distinguishing between limiting and non-limiting cases.	AO2

		4.1.12	Solve equilibrium problems involving friction using $F = \mu R$ or $F \leq \mu R$ as appropriate.	AO2
4.2	Understanding Kinematics	4.2.1	Recognise displacement as a vector and distance as its scalar equivalent.	AO1
		4.2.2	Recognise velocity as the rate of change of displacement (vector) and speed as its scalar equivalent.	AO1
		4.2.3	Recognise acceleration as the rate of change of velocity.	AO1
		4.2.4	Recognise deceleration as the rate of decrease of velocity.	AO1
		4.2.5	Solve problems using: average velocity $= \frac{\Delta s}{\Delta t} = \frac{\text{change in displacement}}{\text{time}}$ and average acceleration $\frac{\Delta v}{\Delta t} = \frac{\text{change in velocity}}{\text{time}}$	AO2
		4.2.6	Apply SUVAT equations: $S = ut + \frac{1}{2}at^2$ $v = u + at$ $v^2 = u^2 + 2aS$ $S = \frac{1}{2}(u + v)t$ to solve constant acceleration problems.	AO2
		4.2.7	Use $g = 9.81 \text{ m/s}^2$ as the standard value of gravitational acceleration (and $g \approx 10 \text{ m/s}^2$ for estimation).	AO1
		4.2.8	Sketch displacement–time and velocity–time graphs.	AO2
		4.2.9	Determine instantaneous velocity from the gradient of a displacement–time graph.	AO2
		4.2.10	Determine total displacement from the area under a velocity–time graph.	AO2
		4.2.11	Determine instantaneous acceleration from the gradient of a velocity–time graph.	AO2
		4.2.12	Apply differentiation and integration with respect to time (Pure 1 knowledge) to model displacement, velocity, and acceleration.	AO2

		4.2.13	Recognise linear momentum as a vector given by $p = mv$.	AO1
		4.2.14	Apply the principle of conservation of linear momentum: total momentum before = total momentum after (if no external force acts).	AO2
		4.2.15	Solve problems involving collisions and explosions using conservation of momentum.	AO2
4.3	Understanding the action of resultant forces	4.3.1	State and apply Newton's First Law: "An object maintains its state of motion unless acted upon by an external resultant force."	AO1
		4.3.2	State and apply Newton's Second Law: an object accelerates when acted upon by an external resultant force.	AO1
		4.3.3	Recognise that Newton's Second Law implies force is proportional to the rate of change of momentum.	AO1
		4.3.4	Solve problems using $F = ma$ and $F = \frac{d(mv)}{dt}$.	AO2
		4.3.5	Recognise weight as the force due to gravity, and calculate it using $W = mg$.	AO1
		4.3.6	Model objects connected by light inextensible strings or light rigid bars as perfect connectors.	AO1
		4.3.7	Model smooth objects or surfaces as having no friction.	AO1
		4.3.8	Solve problems involving motion under constant forces, including motion on inclined planes.	AO2
		4.3.9	Solve problems involving non-constant forces by analysing a specific instant.	AO2
		4.3.10	Solve problems where two particles are connected by a perfect connector.	AO2
		4.3.11	Recognise impulse as the change in momentum, $\Delta p = mv - mu$.	AO1
		4.3.12	Calculate forces in collisions using $F = \frac{\text{Impulse}}{\text{Time Taken}}$.	AO2

		4.3.13	Recognise the coefficient of dynamic friction μ as the frictional ratio when an object is moving.	AO1
		4.3.14	Solve limiting equilibrium problems using $F = \mu R$.	AO2
4.4	Understanding energy, work and power	4.4.1	Recognise that work done is the energy transferred in a process, and that energy is the capacity to do work.	AO1
		4.4.2	Recognise that the work done by a force is proportional to the product of force and displacement in the direction of the force.	AO1
		4.4.3	Solve problems where displacement is not in the direction of the force using $W = Fd \cos \theta$.	AO2
		4.4.4	Recognise and use the kinetic energy formula: $KE = \frac{1}{2}mv^2$.	AO1
		4.4.5	Recognise and use the gravitational potential energy formula: $GPE = mgh$.	AO1
		4.4.6	Recognise and use the elastic potential energy formula: $EPE = \frac{1}{2}kx^2$.	AO1
		4.4.7	Recognise and use conservation of energy: total energy before = total energy after, if no external force acts.	AO1
		4.4.8	Recognise that the geometry of a particle's motion is irrelevant if energy is conserved, only initial and final configurations matter.	AO1
		4.4.9	Solve problems using the principle of conservation of energy.	AO2
		4.4.10	Recognise that power is the rate at which work is done.	AO1
		4.4.11	Solve problems using $P = \frac{W}{t} = Fv$.	AO2

Mechanics 2

Aim: To apply mathematical methods to study more complex types of motion, focusing on two-dimensional movement and objects that cannot be treated as particles.

Note: Knowledge from Pure Mathematics 1 is assumed and may be assessed within this unit, even though it is not listed here.

	The learner will:	SLO #	Assessment Criteria - The learner can:	Cognitive levels
5.1	Understanding projectile motion	5.1.1	Resolve a velocity vector into its horizontal and vertical components.	AO2
		5.1.2	Identify and use the fact that, without air resistance, horizontal acceleration is zero.	AO1
		5.1.3	Identify and use the fact that, without air resistance, vertical acceleration is equal to the acceleration due to gravity.	AO1
		5.1.4	Interpret projectile motion as independent vertical and horizontal components that can be analysed separately.	AO2
		5.1.5	Apply the SUVAT equations independently to vertical and horizontal motion.	AO2
		5.1.6	Solve problems involving projectile motion where the object returns to the same or a lower height than the launch point.	AO2
		5.1.7	Determine distances, speeds, and angles of motion at specific times or positions.	AO2
		5.1.8	Calculate the time of flight, i.e., the total duration between launch and return to the horizontal plane.	AO2
		5.1.9	Calculate the horizontal range, i.e., the distance travelled before the projectile returns to the horizontal plane.	AO2
		5.1.10	Calculate the maximum vertical height of a projectile when its vertical velocity becomes zero.	AO2

		5.1.11	Describe qualitatively how air resistance or varying gravitational acceleration affects projectile motion.	AO3
5.2	Understanding forces on extended objects. (moments and centre of mass)	5.2.1	Interpret a moment as the turning effect of a force, given by the force multiplied by its perpendicular distance from a pivot.	AO1
		5.2.2	Calculate moments using $M = Fd$ or $M = Fd \sin \theta$.	AO2
		5.2.3	Identify and use the fact that moments act in a clockwise or anticlockwise direction.	AO1
		5.2.4	Calculate a resultant moment by summing a set of moments.	AO2
		5.2.5	Determine the magnitude of a force or its distance from a pivot for equilibrium conditions (resultant force and resultant moment equal to zero).	AO2
		5.2.6	Interpret a couple as a pair of equal, anti-parallel forces.	AO1
		5.2.7	Prove that the resultant moment of a couple is equal to the magnitude of one of the forces multiplied by their separation, independent of the centre of rotation.	AO3
		5.2.8	Apply the fact that, with multiple pivots, moments can be taken about any one of them.	AO2
		5.2.9	Apply the fact that if a beam is about to rotate about one pivot, the reaction forces at all other pivots are zero.	AO2
		5.2.10	Interpret the centre of mass of an object as the point where its mass appears to act.	AO1
		5.2.11	Apply the fact that the centre of mass of a uniform bar is at its midpoint.	AO2
		5.2.12	Use moments to determine the centre of mass of a non-uniform bar.	AO2
		5.2.13	Determine the centre of mass of particles on a line using $\sum m_i x_i = \underline{x} \sum m_i$.	AO2
		5.2.14	Prove the relation $\sum m_i x_i = \underline{x} \sum m_i$ for particles on a line.	AO3

		5.2.15	Determine the centre of mass of particles in a plane using $\sum m_i x_i = \underline{x} \sum m_i$ and $\sum m_i y_i = \underline{y} \sum m_i$	AO2
		5.2.16	Find the centre of mass of simple uniform laminas (circle, rectangle, triangle).	AO2
		5.2.17	Find the centre of mass of a composite shape by combining or subtracting simple laminas.	AO2
		5.2.18	Solve equilibrium and dynamics problems involving non-uniform bars and uniform plane laminas.	AO2
5.3	Understanding circular motion and simple harmonic motion.	5.3.1	Interpret angular velocity ω as the angle turned per second (in radians) and use $\omega = \frac{\theta}{t}$.	AO1
		5.3.2	Apply the relationship between linear speed v and angular speed ω for circular motion of radius r , $v = \omega r$.	AO2
		5.3.3	Interpret frequency f as revolutions per second and period T as the time for one revolution.	AO1
		5.3.4	Use the relationship $f = \frac{1}{T} = \frac{\omega}{2\pi}$.	AO2
		5.3.5	Demonstrate that uniform circular motion is accelerated, and establish $a = \frac{v^2}{r} = r\omega^2$.	AO3
		5.3.6	Use Newton's laws to derive the centripetal force $F = \frac{mv^2}{r} = mr\omega^2$, directed to the centre.	AO3
		5.3.7	Calculate quantities such as linear speed, angular speed, centripetal acceleration, centripetal force, or radius of circular motion.	AO2
		5.3.8	Show that the x -coordinate of circular motion about the origin is $x = R \cos(\omega t)$.	AO3
		5.3.9	Differentiate to obtain velocity and acceleration in the x -direction: $v = -R\omega \sin(\omega t)$, $a = -R\omega^2 \cos(\omega t)$.	AO3

		5.3.10	Interpret simple harmonic motion (SHM) as motion under the restoring force $F = -kx$, where x is displacement from equilibrium.	AO1
		5.3.11	Demonstrate that SHM follows the equations $x = A \cos(\omega t)$, $v = -A\omega \sin(\omega t)$, $a = -A\omega^2 \cos(\omega t)$, and link to circular motion with $\omega = \sqrt{\frac{k}{m}}$.	AO3

Probability and Statistics

Aim: To study complex systems by analysing large numbers of similar events through averaging. The aim is to evaluate models of random events and interpret data arising from experiments.

Note: Prior knowledge of Pure Mathematics 1 is assumed and may be assessed within this unit, though it is not restated here.

	The learner will:	SLO #	Assessment Criteria - The learner can:	Cognitive levels
6.1	Understanding probability	6.1.1	Demonstrate awareness that probability measures the likelihood of an event and is expressed between 0 (impossible) and 1 (certain).	AO1
		6.1.2	Demonstrate awareness of the terms experiment, event, outcome, sample space, mutually exclusive, independent, equally likely, and biased in the context of probability.	AO1
		6.1.3	Apply the fact that the sum of probabilities of all possible events equals one.	AO2
		6.1.4	Use $P(event)$ notation to represent probabilities.	AO1
		6.1.5	Calculate probabilities of specific outcomes in simple dice and card experiments (single roll or draw).	AO2
		6.1.6	Use sample space diagrams, Venn diagrams, tree diagrams, and histograms to represent probability spaces and extract probabilities.	AO2
		6.1.7	Demonstrate awareness that the intersection of A and B means “ A and B ,” denoted by $A \cap B$.	AO1
		6.1.8	Demonstrate awareness that the union of A and B means “ A or B ,” denoted by $A \cup B$.	AO1
		6.1.9	Demonstrate awareness that the complement of A means “not A ,” denoted by A' .	AO1

		6.1.10	Apply the rule for mutually exclusive events: $P(A \cup B) = P(A) + P(B)$.	AO2
		6.1.11	Apply the rule for independent events: $P(A \cap B) = P(A) \times P(B)$, and use the multiplication rule to test independence.	AO2
		6.1.12	Use conditional probability notation $P(A B)$ to represent the probability of A given B and $P(A B')$ to represent the probability of A given 'not B'.	AO2
		6.1.13	Apply the fact that for independent events $P(A B) = P(A B') = P(A)$ and $P(B A) = P(B A') = P(B)$, and use them to determine the independence of events.	AO2
		6.1.14	Apply and justify the addition formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, with reasoning using Venn diagrams or other methods.	AO3
		6.1.15	Apply and justify the multiplication formula $P(A \cap B) = P(A B) \times P(B)$, with reasoning using Venn diagrams or other methods.	AO3
		6.1.16	Solve probability problems involving conditional and/or successive events.	AO2
6.2	Understanding representation and characterisation of data	6.2.1	Demonstrate awareness of the difference between qualitative vs. quantitative data, and discrete vs. continuous data, and choose appropriate representations.	AO1
		6.2.2	Construct histograms, box plots, and stem-and-leaf diagrams for data sets, and extract information from them.	AO2
		6.2.3	Use grouped frequency tables to organise data into classes and determine class boundaries, midpoints, and widths.	AO2
		6.2.4	Demonstrate awareness that measures of location describe position in a data set, while measures of central tendency describe the centre.	AO1
		6.2.5	Calculate mean, median, and mode (or modal class) for a set of data.	AO2

		6.2.6	Calculate the combined mean of two data sets using the weighted mean formula: If set A has the size n_a and mean $\underline{x}_a$ and set B has n_b and mean $\underline{x}_b$, then measure the mean of A and B is: $\underline{x} = \frac{n_a \underline{x}_a + n_b \underline{x}_b}{n_a + n_b}$	AO2
		6.2.7	Calculate the mean from a frequency table using $\underline{x} = \frac{\sum xf}{\sum f}$.	AO2
		6.2.8	Determine quartiles and percentiles as measures of location for a set of data.	AO2
		6.2.9	Apply interpolation to estimate the median, quartiles, and percentiles from grouped frequency tables.	AO2
		6.2.10	Determine quartiles from grouped continuous data or cumulative frequency tables.	AO2
		6.2.11	Demonstrate awareness that measures of spread describe the distribution of data as a single value.	AO1
		6.2.12	Calculate range, interquartile range (IQR), and interpercentile range (IPR).	AO2
		6.2.13	Calculate the summary statistic: $S_{xx} = \sum (x - \underline{x})^2 = \sum x^2 - \frac{(\sum x)^2}{n}$.	AO2
		6.2.14	Calculate variance using $\sigma^2 = \frac{S_{xx}}{n}$ as a measure of spread.	AO2
		6.2.15	Calculate standard deviation using $\sigma = \sqrt{\frac{S_{xx}}{n}}$	AO2
		6.2.16	Apply data coding of the form $y = \frac{(x-a)}{b}$.	AO2
		6.2.17	Calculate the mean and standard deviation of coded data, and convert back to the original mean $\underline{x} = b\underline{y} + a$ and standard deviation $\sigma_x = b\sigma_y$.	AO2
		6.2.18	Identify outliers using given definitions or impossibility arguments, and clean the data.	AO3

		6.2.19	Identify positive skew, negative skew, or symmetry in data from frequency tables, bar charts, or box plots (without calculation).	AO1
		6.2.20	Compare two data sets using measures of location, spread, and skewness.	AO3
6.3	Understanding discrete random variables	6.3.1	Recognise that a discrete random variable takes only specific numerical values, with outcomes unknown before the experiment (e.g. dice roll).	AO1
		6.3.2	Recognise that a probability distribution is uniform if each outcome is equally likely (e.g. fair dice).	AO1
		6.3.3	Use probability notation for random variables, $P(X = x)$.	AO1
		6.3.4	Construct tables or diagrams to represent the probability distribution of a discrete random variable.	AO2
		6.3.5	Calculate the cumulative distribution function $F(x) = P(X \leq x)$.	AO2
		6.3.6	Construct cumulative distribution tables for discrete random variables.	AO2
		6.3.7	Calculate the expected value of a discrete random variable, $E(X) = \sum xP(X = x)$.	AO2
		6.3.8	Calculate the variance of a discrete random variable, $Var(X) = E[(X - E(X))^2] = E(X^2) - (E(X))^2$.	AO2
		6.3.9	Recognise that if X is a discrete random variable, then $f(X)$ is also a discrete random variable.	AO1
		6.3.10	Calculate the expected value of a function, $E(f(X)) = \sum f(X)P(X = x)$.	AO2
		6.3.11	Use expectation properties: $E(aX + b) = aE(X) + b$ and $E(X + Y) = E(X) + E(Y)$.	AO1
		6.3.12	Use variance property: $Var(aX + b) = a^2Var(X)$.	AO1
		6.3.13	Use the results for a uniform distribution over $[1, 2, 3, \dots, n]$: $E(X) = \frac{n+1}{2}$, $Var(X) = \frac{(n+1)(n-1)}{12}$.	AO2

6.4	Understanding the normal distribution	6.4.1	Recognise that a continuous random variable can take any value within a range, and that $P(X = x) = 0$.	AO1
		6.4.2	Use the fact that probability in a continuous distribution is given by the area under the curve, with total area = 1.	AO1
		6.4.3	Recall properties of the Normal Distribution: defined by μ, σ^2 ; symmetric; mean=median=mode; bell-shaped; inflection point at $\mu \pm \sigma$; empirical rule (68% – 95% – 99.75%).	AO1
		6.4.4	Use the notation $X \sim N(\mu, \sigma^2)$ for a normally distributed variable.	AO1
		6.4.5	Recall that the Standard Normal Distribution is $Z \sim N(0, 1^2)$.	AO1
		6.4.6	Apply standardisation: $Z = \frac{X - \mu}{\sigma}$, and use it to find or infer unknown means/variances.	AO2
		6.4.7	Use diagrams or Normal tables to find probabilities, or determine values of X given $P(Z > z) = p$.	AO2
6.5	Understanding correlation and regression	6.5.1	Identify bivariate data as pairs of values of two variables.	AO1
		6.5.2	Plot bivariate data on scatter diagrams and interpret patterns.	AO2
		6.5.3	Distinguish between the independent (explanatory) variable on the x-axis and the dependent (response) variable on the y-axis.	AO1
		6.5.4	Interpret correlation as the linear relationship between two variables, recognising its direction and strength.	AO1
		6.5.5	Express linear regression as the regression line of y on x , of the form $y = ax + b$.	AO1

		6.5.6	Calculate summary statistics: $S_{xx} = \sum (x - \underline{x})^2 = \sum x^2 - \frac{(\sum x)^2}{n},$ $S_{yy} = \sum (y - \underline{y})^2 = \sum y^2 - \frac{(\sum y)^2}{n},$ $S_{xy} = \sum xy - \frac{\sum x \sum y}{n}.$	AO2
		6.5.7	Calculate the least squares regression line using $a = \frac{S_{xy}}{S_{xx}}$, $b = \underline{y} - a\underline{x}$, and apply it for interpolation.	AO2
		6.5.8	Recognise the product moment correlation coefficient (PMCC) as a measure of strength of linear association.	AO1
		6.5.9	Calculate PMCC using $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$.	AO2
		6.5.10	Apply coding to one or both variables and confirm that PMCC remains unchanged.	AO2
		6.5.11	Draw conclusions from regression and correlation, and apply extrapolation to estimate values outside the data range.	AO3

Formula Booklet

1. Algebra

a. Basic Algebraic Identities

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = a^2 - b^2$$

b. Laws of indices

$$a^m \cdot a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^0 = 1 \text{ (for } a \neq 0 \text{)}$$

c. Laws of Logarithms

$$\log_a(xy) = \log_a(x) + \log_a(y)$$

$$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$$

$$\log_a(x^n) = n \log_a(x)$$

$$\log_a(b) = \frac{\log_c(b)}{\log_c(a)}$$

d. Quadratic Equations

$$\text{Standard form: } ax^2 + bx + c = 0$$

$$\text{Quadratic formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{Completing the square: If } p(x + q)^2 + r = 0, \text{ then } x = -q \pm \sqrt{\frac{-r}{p}}$$

e. Arithmetic Progression & series

$$u_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}(a + l) = \frac{n}{2}(a + u_n)$$

f. Geometric Progression & series

$$u_n = ar^{n-1}$$
$$S_n = \frac{a(1 - r^n)}{1 - r} \text{ where } (r \neq 1)$$

$$S_\infty = \frac{a}{1 - r} \text{ where } (|r| < 1)$$

g. Binomial Expansion

$$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^k a^{n-k}$$

h. Summation formulae

$$\sum_{r=1}^n r = \frac{n(n+1)}{2}$$
$$\sum_{r=1}^n r^2 = \frac{n(n+1)(2n+1)}{6}$$
$$\sum_{r=1}^n r^3 = \left(\frac{n(n+1)}{2}\right)^2$$

i. Modulus properties

$$|a| = |b| \Leftrightarrow a^2 = b^2$$
$$|x - a| < b \Leftrightarrow a - b < x < a + b$$

2. Geometry

a. Circle properties

$$\text{Arc Length: } s = r\theta$$

$$\text{Sector Area: } A = \frac{1}{2}r^2\theta$$

b. Triangles

$$\text{Area: } A = \frac{1}{2} ab \sin C$$

$$\text{Sine rule: } \frac{a}{\sin A} + \frac{b}{\sin B} + \frac{c}{\sin C}$$

$$\text{Cosine rule: } a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Pythagorean Theorem: } a^2 = b^2 + c^2 \text{ (in a right triangle)}$$

c. 3D solids

$$\text{Surface area of a sphere: } A = 4\pi r^2$$

$$\text{Volume of a sphere: } V = \frac{4}{3}\pi r^3$$

$$\text{Surface area of a cone: } A = \pi r l + \pi r^2$$

$$\text{Volume of a cone: } V = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a pyramid: } V = \frac{1}{3} \times \text{base area} \times h$$

$$\text{Surface area of a cylinder: } A = 2\pi r h + 2\pi r^2$$

$$\text{Volume of a cylinder: } V = \pi r^2 h$$

3. Trigonometric Identities

a. Basic Definitions

$$\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$$

b. Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta \equiv 1$$

$$\sec^2 \theta \equiv 1 + \tan^2 \theta$$

$$\csc^2 \theta \equiv 1 + \cot^2 \theta$$

c. Compound angle identities

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \text{ where } (A \pm B) \neq \left(n + \frac{1}{2}\right)\pi$$

d. Double angle identities

$$\sin(2A) \equiv 2 \sin A \cos A$$

$$\cos(2A) \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$$

$$\tan(2A) \equiv \frac{2\tan A}{1 - \tan^2 A} \text{ where } (2A) \neq \left(n + \frac{1}{2}\right)\pi$$

e. Sum-to-product identities

$$\sin A \pm \sin B \equiv 2 \sin \frac{1}{2}(A \pm B) \cos \frac{1}{2}(A \mp B)$$

$$\cos A + \cos B \equiv 2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$$

$$\cos A - \cos B \equiv -2 \cos \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$$

f. R-formula (Linear combination of Sine and Cosine)

$$a \sin x \pm b \cos x \equiv R \sin(x \pm \alpha)$$

$$a \cos x \pm b \sin x \equiv R \cos(x \mp \alpha)$$

$$\text{where } R = \sqrt{a^2 + b^2}, \quad R \cos \alpha = a, \quad R \sin \alpha = b \quad \left(R > 0, 0 < \alpha < \frac{\pi}{2}\right)$$

4. Differentiation

a. Basic functions

$$\frac{d}{dx}(x)^n = nx^{n-1} \quad (n \in \mathbb{R})$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad (x > 0)$$

b. Trigonometric functions

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

c. Hyperbolic functions

$$\frac{d}{dx}(\sinh x) = \cosh x$$

$$\frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$

d. Inverse Hyperbolic functions

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$$

$$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}, \quad x > 1$$

$$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}, \quad |x| < 1$$

e. Rules of differentiation

$$\text{Product Rule: } \frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

$$\text{Quotient Rule: } \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$\text{Chain Rule: } \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

5. Integration

a. Basic Integrals

$$\int f(x)dx = F(x) + C, \quad (\text{where } F'(x) = f(x))$$

$$\sinh \int x \, dx = \cosh x + C$$

$$\cosh \int x \, dx = \sinh x + C$$

$$\int \tanh x \, dx = \ln (\cosh x) + C$$

b. Rational forms

$$\int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C$$

$$\int \frac{1}{x^2 - a^2} \, dx = \frac{1}{2a} \ln \left(\frac{x - a}{x + a} \right) + C, \quad (x > a)$$

$$\int \frac{1}{a^2 - x^2} \, dx = \frac{1}{2a} \ln \left(\frac{a + x}{a - x} \right) + C, \quad (|x| < a)$$

c. Root forms

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \sinh^{-1} \left(\frac{x}{a} \right) + C = \ln(x + \sqrt{x^2 + a^2}) + C, \quad (x > a)$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \cosh^{-1} \left(\frac{x}{a} \right) + C = \ln(x + \sqrt{x^2 - a^2}) + C, \quad (x > a) \quad (x > a)$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \left(\frac{x}{a} \right) + C, \quad (|x| < a)$$

d. Numerical Integration (Trapezium Rule)

$$\int_a^b y \, dx \approx \frac{1}{2} h [y_0 + 2(y_1 + y_2 + \cdots + y_{n-1}) + y_n]$$

$$\text{where,} \quad h = \frac{b - a}{n}, \quad y_i = f(a + ih)$$

6. Mechanics

a. Kinematics

$$\text{Average velocity: } v_{avg} = \frac{\Delta s}{\Delta t}$$

$$\text{Average acceleration: } a_{avg} = \frac{\Delta v}{\Delta t}$$

Equations of uniformly accelerated motion:

$$s = ut + \frac{1}{2}at^2$$

$$v = u + at$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

b. Dynamics

$$\text{Newton's second law: } F = ma = \frac{d(mv)}{dt}$$

$$\text{Impulse: } F \times \Delta t = \Delta(mv)$$

$$\text{Friction: } F \leq \mu R$$

c. Work, Energy and Power

$$\text{Work: } W = Fd\cos\theta$$

$$\text{Power: } P = \frac{W}{t} = Fv$$

d. Moments

$$\text{Moment} = Fd\sin\theta$$

e. Circular motion

$$\text{Centripetal acceleration: } a = \frac{v^2}{r} = r\omega^2$$

$$\text{Centripetal force: } F = \frac{mv^2}{r} = mr\omega^2$$

f. Centre of mass of uniform of bodies

Triangular lamina: $\frac{2}{3}$ *along median from vertex*

Solid hemisphere of radius r : $\frac{3}{8}r$ *from centre*

Hemispherical shell of radius r : $\frac{1}{2}r$ *from centre*

Circular arc of radius r , angle 2α : $\frac{r\sin\alpha}{\alpha}$ *from centre*

Circular sector of radius r , angle 2α : $\frac{2r\sin\alpha}{3\alpha}$ *from centre*

Solid cone/pyramid of height h : $\frac{3}{4}h$ *above the base on the line from centre of base to vertex*

Conical shell of height h : $\frac{1}{3}h$ *above the base on the line from centre of base to vertex*

7. Probability and Statistics

a. Mean of two sets

$$\bar{x} = \frac{n_a \bar{x}_a + n_b \bar{x}_b}{n_a + n_b}$$

where n_i is the size of the set i , and $\bar{x}_i$ is the mean of set i .

b. Mean from a frequency table

$$\bar{x} = \frac{\sum xf}{\sum f}$$

Where f is the frequency of value x .

c. Variance and Standard Deviation

Variance:

$$\sigma^2 = \frac{S_{xx}}{n}$$

Standard Deviation:

$$\sigma = \sqrt{\frac{S_{xx}}{n}}$$

d. Summary Statistics

$$\begin{aligned} S_{xx} &= \sum x^2 - \frac{(\sum x)^2}{n} \\ S_{yy} &= \sum y^2 - \frac{(\sum y)^2}{n} \\ S_{xy} &= \sum xy - \frac{\sum x \sum y}{n} \end{aligned}$$

e. Least Squares Linear Regression

Line of y on x :

$$y = ax + b$$

Where,

$$a = \frac{S_{xy}}{S_{xx}}, \quad b = \bar{y} - a\bar{x}$$

f. Product Moment Correlation

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$$

g. Coding (if $y = \frac{x-a}{b}$):

$$b\bar{y} + a = \bar{x}, \quad \sigma_x = b\sigma_y$$

h. Expected value of a Discrete Random Variable

$$E(X) = \sum xP(X = x)$$

i. Variance of a Discrete Random Variable

$$Var(X) = E[(X - E(X))^2] = E(X^2) - (E(X))^2$$

8. Normal Distribution $N(\mu, \sigma^2)$

The probability density function of a normal distribution with mean μ and variance σ^2 is:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If Z is a standard normal distribution variable ($\mu = 0, \sigma^2 = 1$), then the cumulative distribution function is:

$$\Phi(z) = P(Z < z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-\frac{t^2}{2}} dt$$

For **negative values** of z , we use:

$$\Phi(-z) = 1 - \Phi(z)$$

The table on the next page gives $\Phi(z)$ for $z \geq 0$.

To use: take the row for the first two decimals of z , then add the column header for the third decimal place.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	11	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	8	11	15	19	23	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	5	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	5	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	19	22	25	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	15	19	22	23
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	5	8	10	13	15	18	20	23	25
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	9	12	14	16	19	21	24
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	9	12	14	15	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	5	9	10	13	15	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	1	3	5	6	7	9	10	11	13
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	2	4	5	6	7	8	9	11
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	0	2	3	4	5	6	7	8	9
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	0	1	2	3	4	5	6	7	8
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	0	1	1	2	3	4	4	5	6
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	0	1	1	2	2	3	3	4	4
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	0	0	1	1	2	2	3	3	4
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	0	1	1	1	2	2	3	3
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	0	0	1	1	1	2	2	2
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	0	0	1	1	1	1	2	2
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	0	0	0	1	1	1	1	1
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	0	0	1	1	1	1	1
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	0	0	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	0	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	0	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	1

If $Z \sim N(0,1)$, then the z-values corresponding to common cumulative probabilities $P(Z < z) = p$ are:

P	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
Z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291